Calculus

Week # 2

AP CALCULUS AB
FINAL CHAPTERS 1-3 PART I

DIRECTIONS:

Show work on a separate sheet of paper. Make sure to pace yourself so you can finish the entire exam. Leave all answers in radical form. **No calculator** may be used on this portion of the exam.

The following problems are worth 8 points each.

1. Find $\lim_{x\to 4} f(x) = \dfrac{x^2 - x - 12}{x^2 - 16}$

A. 0
B. $\frac{7}{8}$
C. 4
D. 8

2. Find $\lim_{x\to 0} f(x) = \dfrac{\sin 3x}{x}$

A. 3
B. 1
C. 0
D. ∞

3. Find $\lim_{x\to 5^+} f(x) = \dfrac{-1}{x-5}$

A. -∞
B. 0
C. -1
D. ∞

4. Find $\lim_{x\to 7} f(x) = \begin{cases} 3x+1 & if\ x = 7 \\ 2x & if\ x \neq 7 \end{cases}$

A. 22
B. 14
C. 0
D. No Limit

5. Find $\lim_{x\to -4^-} \|x\| - 2$

A. -7
B. -6
C. -2
D. No Limit

6. Write the equation of the line tangent to the graph of $f(x) = 3x^2 + \dfrac{1}{x}$ at the point (1, 4).

A. y = 5x-1
B. y = 5x-9
C. y = 4x+8
D. y = 4x

7. Find the derivative of $f(x) = \sin^4 5x$

A. $4\cos^3 5x dx$
C. $20\sin^3 5x \cos 5x dx$
B. $20\cos^3 5x dx$
D. $20\sin^3 5x dx$

8. Find the value of c so that the function $f(x) = \begin{cases} 3x^2 - 1 & \text{if } x < 3 \\ \frac{cx+7}{2} & \text{if } x \geq 3 \end{cases}$ so it is continuous everywhere.

A. 42.5
C. 15
B. 26
D. 3

9. If $f(x) = \frac{x+3}{x^2+1}$, then f' (-2) =

A. 9/25
C. $\frac{1}{25}$
B. $-\frac{1}{4}$
D. $\frac{1}{4}$

10. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

(A) $-\cos(e^{-x})$
(B) $\cos(e^{-x}) + e^{-x}$
(C) $\cos(e^{-x}) - e^{-x}$
(D) $e^{-x}\cos(e^{-x})$
(E) $-e^{-x}\cos(e^{-x})$

11. Find the derivative of $y = 5(2x+3)^6$

A. y' = $10(2x+3)^6$
C. y' = $60(2x+3)^5$
B. y' = $30(2x+3)^5$
D. y' = $30(2x+3)^6$

12. A particle moves along the x-axis so that its position at time t > 0 is given by $s(t) = t^2 - 8t + 7$. At what time t > 0 will the velocity of the particle be zero?

A. t = 2
C. t = 4
B. t = 3
D. t = 7

13. Find the value of c guaranteed by the Mean Value theorem for the function f(x) = 9/x, over the interval [1, 9].

A. 3 C. -3

B. 0 D. Not Possible

14. What is the instantaneous rate of change of y = -cos x at the point $\frac{\pi}{6}$?

A. $-\frac{1}{\sqrt{3}}$ C. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{2}$ D. $\sqrt{3}$

15. Use logarithmic differentiation to find the derivative of $y = 2^{\sin x}$

A. $\frac{dy}{dx} = \ln 2 \cdot \cos x$ C. $\frac{dy}{dx} = 2^{\sin x} \cdot \cos x$

B. $\frac{dy}{dx} = 2^{\sin x} \cdot \ln 2 \cdot \cos x$ D. $\frac{dy}{dx} = 2^{\sin x}\left[\ln 2 \cdot \cos x + \frac{1}{2}\sin x\right]$

E. none of these

16. Find the horizontal asymptote for $f(x) = \frac{3x^2 + 2x - 16}{x^2 - 7}$.

A. $x = \pm\sqrt{7}$ C. y = 3x-7

B. y = 3 D. y = 0

17. Use implicit differentiation to find the derivative of $5x^2 - 2xy + 7y^2 = 0$.

A. $y' = \frac{y-5x}{7y-x}$ C. $y' = \frac{5y+7x}{x}$

B. $y' = \frac{y-5x}{7y}$ D. y’ = 5x+7y

18. Find f’(x) when $f(x) = \sin^3 4x$

A. $4\cos^3 4x$ C. $12(\sin^2 4x)(\cos 4x)$

B. $3(\sin^2 4x)(\cos 4x)$ D. $\cos^3 4x$

E. None of these

19. Find the domain and range of the function $f(x) = \sqrt{2x} + 3$

A. D: [0, ∞) and R: (0, ∞)
C. D: [0, ∞) and R: [0, ∞)

B. D: [0, ∞) and R: [3, ∞)
D. D: [0, ∞) and R: (3, ∞)

Use the following information to answer questions 20-22.

x	f	g	f'	g'
1	**3**	**5**	**8**	**-2**
2	**-4**	**7**	**-9**	**3**
3	**5**	**2**	**4**	**-6**

20. Find the derivative of 3f at x = 2.

A. -12
C. -9
B. -4
D. -27

21. Find the derivative of $\frac{f}{g}$ at x = 1.

A. $\frac{46}{25}$
C. 2

B. $\frac{34}{25}$
D. -4

22. Find the derivative of 4gf at x = 3.

A. -22
C. -88
B. -38
D. -152

23. Given the f(2) = -5, and f'(2) = 4, write the tangent line for f.

A. y = 4x+22
C. y = 4x-3

B. y = 4x-13
D. y = -5x+14

24. Use your answer to #23 to approximate f(2.2)

A. 11.0
C. 3

B. 5.8
D. -4.2

DIRECTIONS:

Show any work on a separate sheet of paper (if needed). Any decimal answer should be rounded off to the third significant decimal place. ***GOOD LUCK!!!***

The following problem is worth 40 points.

25. Use the function $f(x) = x^3 - 12x + 20$ to answer the following questions.

a. Find the intervals where the f is increasing or decreasing.

b. Find the relative extrema of f use the first derivative test to tell if it is a maximum or minimum.

c. Find any points of inflection

d. Find the intervals where the graph of f is concave up or concave down.

e. Using the information above accurately sketch the graph of f.

The following problem is worth 8 points.

26. $y = xe^2 - e^x$

The following problems are worth 10 points each.

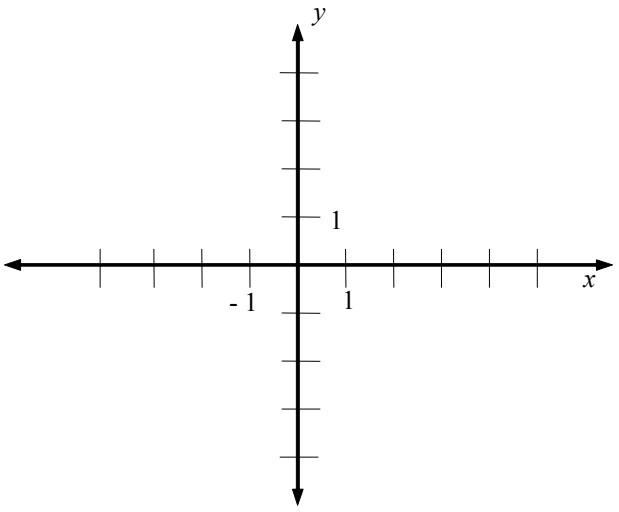
27. Find d^2y/dx^2 for the function $x^2+y^2 = 25$.

28. Determine the x-coordinate where the function y = sin 2x has a horizontal tangent in the interval $[0, \pi]$.

29. Given the function $y = x^2 + 3x$ find the derivative using the definition.

30. Sketch the graph of a **continuous** function $f(x)$ that satisfies the following conditions.

- $f(-2) = -3, \quad \lim_{x \to 2} f(x) = 4$
- $f'(-2) = 0, \quad f'(2) = 0$
- $f'(x) < 0$ if $x < -2$ or $x > 2$, $\quad f'(x) > 0$ if $-2 < x < 2$
- $f''(x) < 0$ if $x < -3$ or $x > 0$, $\quad f''(x) > 0$ if $-3 < x < 0$
- $\lim_{x \to -\infty} f(x) = 0$



The Following problem is worth 20 points.

31. A tennis ball is thrown so that its position at any time $t \geq 0$ is given by $s(t) = -16t^2 + 192t + 208$.

a. The velocity of the tennis ball at any time.
b. When will the tennis ball hit the ground?
c. What is the maximum height of the tennis ball?
d. When will the ball reach its maximum height?
e. What is the velocity of the tennis ball when it hits the ground?